This is the March 2011 version of the High School Mathematics Model Curriculum for the conceptual category Geometry. (Note: The conceptual categories Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability do not directly align with high school mathematics courses.) The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Geometry	
Domain	Cluster
Congruence	Experiment with transformations in the plane Understand congruence in terms of rigid motions Prove geometric theorems Make geometric constructions.
Similarity, Right Triangles, and Trigonometry	Understand similarity in terms of similarity transformations Prove theorems involving similarity Define trigonometric ratios and solve problems involving right triangles (+) Apply trigonometry to general triangles
Circles	Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles.
Expressing Geometric Properties with Equations	Translate between the geometric description and the equation for a conic section. Use coordinates to prove simple geometric theorems algebraically.
Geometric Measurement and Dimension	Explain volume formulas and use them to solve problems. Visualize relationships between 2-dimensional and 3-dimensional objects
Modeling with Geometry	Apply geometric concepts in modeling situations.

High School Conceptual Category: Geometry

Domain	**Congruence**
Cluster	***Experiment with transformations in the plane***
Standards	1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. 2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). 3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. 4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. 5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Review vocabulary associated with transformations (e.g. point, line, segment, angle, circle, polygon, parallelogram, perpendicular, rotation reflection, translation).

Provide both individual and small-group activities, allowing adequate time for students to explore and verify conjectures about transformations and develop precise definitions of rotations, reflections and translations.

Provide real-world examples of rigid motions (e.g. Ferris wheels for rotation; mirrors for reflection; moving vehicles for translation).

Use graph paper, transparencies, tracing paper or dynamic geometry software to obtain images of a given figure under specified transformations.

Provide students with a pre-image and a final, transformed image, and ask them to describe the steps required to generate the final image. Show examples with more than one answer (e.g., a reflection might result in the same image as a translation).

Work backwards to determine a sequence of transformations that will carry (map) one figure onto another of the same size and shape.

Focus attention on the attributes (e.g. distances or angle measures) of a geometric figure that remain constant under various transformations.

Make the transition from transformations as physical motions to functions that take points in the plane as inputs and give other points as outputs. The correspondence between the initial and final points determines the transformation.

Analyze various figures (e.g. regular polygons, folk art designs or product logos) to determine which rotations and reflections carry (map) the figure onto itself. These transformations are the “symmetries” of the figure.

Emphasize the importance of understanding a transformation as the correspondence between initial and final points, rather than the physical motion.

Instructional Resources/Tools

Tracing paper (patty paper)
Transparencies
Graph paper
Ruler
Protractor
Computer dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).

Common Misconceptions

The terms "mapping" and "under" are used in special ways when studying transformations.A translation is a type of transformation that moves all the points in the object in a straight line in the same direction. Students should know that not every transformation is a translation.

Students sometimes confuse the terms "transformation" and "translation."

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Specific strategies for mathematics may include:

Use a variety of means to represent rigid motions, including physical manipulatives, coordinate methods, and dynamic geometry software.

Connections:

Rotations, reflections and translations are developed experimentally in the Grade 8, and this experience should be built upon in high school, giving greater attention to precise definitions and formal reasoning.

Transformations can be studied in terms of functions, where the inputs and outputs are points in the plane, rather than numbers.

Rotations are studied again in the cluster about circles.

Domain	Congruence
Cluster	***Understand congruence in terms of rigid motions***
Standards	6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. 7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. 8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Develop the relationship between transformations and congruency. Allow adequate time and provide hands-on activities for students to visually and physically explore rigid motions and congruence.

Use graph paper, tracing paper or dynamic geometry software to obtain images of a given figure under specified rigid motions. Note that size and shape are preserved.

Use rigid motions (translations, reflections and rotations) to determine if two figures are congruent. Compare a given triangle and its image to verify that corresponding sides and corresponding angles are congruent.

Work backwards – given two figures that have the same size and shape, find a sequence of rigid motions that will map one onto the other.

Build on previous learning of transformations and congruency to develop a formal criterion for proving the congruency of triangles. Construct pairs of triangles that satisfy the ASA, SAS or SSS congruence criteria, and use rigid motions to verify that they satisfy the definition of congruent figures. Investigate rigid motions and congruence both algebraically (using coordinates) and logically (using proofs).

Instructional Resources/Tools

Tracing paper (patty paper)
Graph paper
Ruler
Protractor
Computer dynamic geometry software (Geometer's Sketchpad®, Cabri® or Geogebra®); websites with similar tools (such as the National Library of Virtual Manipulatives that features applets for exploring triangle congruence).
Graphing calculators and other handheld technology such as TI-Nspire™.

Common Misconceptions

Some students may believe:

That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception.

That all transformations, including dilation, are rigid motions. Provide counterexamples of this misconception.

That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid.

That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent.

Diverse Learners

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Connections

An understanding of congruence using physical models, transparencies or geometry software is developed in Grade 8, and should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

Domain	Congruence
Cluster	***Prove geometric theorems***
Standards	9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* 10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* 11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Classroom teachers and mathematics education researchers agree that students have a hard time learning how to do geometric proofs. An article by Battista and Clements (1995) (http://investigations.terc.edu/library/bookpapers/geometryand_proof.cfm) provides information for teachers to help students who struggle learn to do proof. The most significant implication for instructional strategies for proof is stated in their conclusion.

> "Ironically, the most effective path to engendering meaningful use of proof in secondary school geometry is to avoid formal proof for much of students' work. By focusing instead on justifying ideas while helping students build the visual and empirical foundation for higher levels of geometric thought, we can lead students to appreciate the need for formal proof. Only then will we be able to use it meaningfully as a mechanism for justifying ideas."

The article and ideas from Niven (1987) offers a few suggestions about teaching proof in geometry:

- Initial geometric understandings and ideas should be taught "without excessive emphasis on rigor." Develop basic geometric ideas outside an axiomatic framework, and then let the importance of the framework (and the framework itself) emerges from the geometry.
- Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between geometric objects should be central to any geometric study and certainly to proof. Battista and Clement make a powerful argument that the use of dynamic geometry software can be an important tool for helping students understand proof.
- "[A]void the deadly elaboration of the obvious" (Niven, p. 43). Often textbooks begin the treatment of formal proof with "easy" proofs, which appear to students to need no proof at all. After presenting many opportunities for students to "justify" properties of geometric figures, formal proof activities should begin with non-obvious conjectures.
- Use the history of geometry and real-world applications to help students develop conceptual understandings before they begin to use formal proof.

Proofs in high school geometry should not be restricted to the two-column format. Most proofs at the college level are done in paragraph form, with the writer explaining and defending a conjecture. In many cases, the two-column format can hinder the student from making sense of the geometry by paying too much attention to format rather than mathematical reasoning.

Some of the theorems listed in this cluster (e.g. the ones about alternate interior angles and the angle sum of a triangle) are logically equivalent to the Euclidean parallel postulate, and this should be acknowledged.

Use dynamic geometry software to allow students to make conjectures that can, in turn, be formally proven. For example, students might notice that the base angles of an isosceles triangle always appear to be congruent when manipulating triangles on the computer screen and could then engage in a more formal discussion of why this occurs.

Common Core Standards Appendix A states, "Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words.

Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning" (p. 29). Different methods of proof will appeal to different learning styles in the classroom.

Instructional Resources/Tools

Dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).

Principles and Standards for School Mathematics, pp.309-318, 342-346.

Niven, Ivan, "Can Geometry Survive in the Secondary Curriculum?" *Learning and Teaching Geometry, K-12.* 1987 Yearbook of the National Council of Teachers of Mathematics

Pythagorean Puzzle - http://www.nsa.gov/academia/_files/collected_learning/high_school/geometry/pythagorean_puz

Though focused on the Pythagorean theorem, this site provides the kind of hands-on experience that should be a precursor to formal proof. In this self-guided investigation, students use Geometer's Sketchpad® to construct a right triangle and discover a geometric proof of the Pythagorean theorem. Students then test the geometric proof with acute and obtuse triangles.

Common Misconceptions

Research over the last four decades suggests that student misconceptions about proof abound:

- even after proving a generalization, students believe that exceptions to the generalization might exist;
- one counterexample is not sufficient;
- the converse of a statement is true (parallel lines do not intersect, lines that do not intersect are parallel); and
- a conjecture is true because it worked in all examples that were explored.

Each of these misconceptions needs to be addressed, both by the ways in which formal proof is taught in geometry and how ideas about "justification" are developed throughout a student's mathematical education.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Specific strategies for mathematics may include:

There is research evidence to suggest that a student's success with proof is correlated with his or her position on the scale of van Hiele levels. Therefore, the best differentiation can be provided by determining at which levels students are operating and providing the scaffolding experiences necessary to help them progress gradually through the levels.

Connections:

Properties of lines and angles, triangles and parallelograms were investigated in Grades 7 and 8. In high school, these properties are revisited in a more formal setting, giving greater attention to precise statements of theorems and establishing these theorems by means of formal reasoning.

The theorem about the midline of a triangle can easily be connected to a unit on similarity. The proof of it is usually based on the similarity property that corresponding sides of similar triangles are proportional.

Domain	**Congruence**
Cluster	***Make geometric constructions***
Standards	12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* 13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Students should analyze each listed construction in terms of what simpler constructions are involved (e.g., constructing parallel lines can be done with two different constructions of perpendicular lines).

Using congruence theorems, ask students to prove that the constructions are correct.

Provide meaningful problems (e.g. constructing the centroid or the incenter of a triangle) to offer students practice in executing basic constructions.

Challenge students to perform the same construction using a compass and string. Use paper folding to produce a reflection; use bisections to produce reflections.

Ask students to write “how-to” manuals, giving verbal instructions for a particular construction.
Offer opportunities for hands-on practice using various construction tools and methods.

Compare dynamic geometry commands to sequences of compass-and-straightedge steps.

Prove, using congruence theorems, that the constructions are correct.

Instructional Resources/Tools

Compass
Straightedge
String
Origami paper
Reflection tool (e.g. Mira®).
Dynamic geometry software (e.g. Geometer’s Sketchpad®, Cabri®, or Geogebra®).

http://www.nsa.gov/academia/_files/collected_learning/high_school/geometry/pythagorean_puzzle.pdf
In this self-guided investigation, students use Geometer’s Sketchpad to construct a right triangle and discover a geometric proof of the Pythagorean Theorem. Students test the geometric proof with acute and obtuse triangles.

http://www.nsa.gov/academia/_files/collected_learning/high_school/geometry/concurrent_events.pdf
This lesson enhances student knowledge of how to use Geometer's Sketchpad to explore geometric concepts (e.g. the points of concurrency in a triangle). It includes the construction of line segments, triangles, circles, perpendicular bisectors of line segments, angle bisectors, altitudes of triangles, and medians of triangles.

http://mathforum.org/alejandre/circles.html
Students will construct a number of compass-and-straightedge designs using ideas from this site.

Common Misconceptions
Some students may believe that a construction is the same as a sketch or drawing. Emphasize the need for precision and accuracy when doing constructions. Stress the idea that a compass and straightedge are identical to a protractor and ruler. Explain the difference between measurement and construction.

Diverse Learners
Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

Drawing geometric shapes with rulers, protractors and technology is developed in Grade 7. In high school, students perform formal geometry constructions using a variety of tools. Students will utilize proofs to justify validity of their constructions.

Domain	Similarity, Right Triangles, and Trigonometry
Cluster	*Understand similarity in terms of similarity transformations*
Standards	1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. 2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. 3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Allow adequate time and hands-on activities for students to explore dilations visually and physically.

Use graph paper and rulers or dynamic geometry software to obtain images of a given figure under dilations having specified centers and scale factors. Carefully observe the images of lines passing through the center of dilation and those not passing through the center, respectively. A line segment passing through the center of dilation will simply be shortened or elongated but will lie on the same line, while the dilation of a line segment that does not pass through the center will be parallel to the original segment (this is intended as a clarification of Standard 1a).

Illustrate two-dimensional dilations using scale drawings and photocopies.

Measure the corresponding angles and sides of the original figure and its image to verify that the corresponding angles are congruent and the corresponding sides are proportional (i.e. stretched or shrunk by the same scale factor). Investigate the SAS and SSS criteria for similar triangles.

Use graph paper and rulers or dynamic geometry software to obtain the image of a given figure under a combination of a dilation followed by a sequence of rigid motions (or rigid motions followed by dilation).

Work backwards – given two similar figures that are related by dilation, determine the center of dilation and scale factor. Given two similar figures that are related by a dilation followed by a sequence of rigid motions, determine the parameters of the dilation and rigid motions that will map one onto the other.

Using the theorem that the angle sum of a triangle is 180°, verify that the AA criterion is equivalent to the AAA criterion.

Given two triangles for which AA holds, use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.

Instructional Resources/Tools

Dot paper
Graph paper
Rulers

Protractors
Pantograph
Photocopy machine
Computer dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).
Web-based applets that demonstrate dilations, such as those at the National Library of Virtual Manipulatives.
Video: *Similarity* by Project Mathematics! (**www.projectmathematics.com)**

Common Misconceptions
Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency.

Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor.

Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.

Diverse Learners
Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Specific strategies for mathematics may include:
Students may be interested in scale models or experiences with blueprints and scale drawings (perhaps in a work related situation) to illustrate similarity.

Connections: Dilations and similarity, including the AA criterion, are investigated in Grade 8, and these experiences should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

Domain	Similarity, Right Triangle, and Trigonometry
Cluster	*Prove theorems involving similarity*
Standards	4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* 5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Review triangle congruence criteria and similarity criteria, if it has already been established.
Review the angle sum theorem for triangles, the alternate interior angle theorem and its converse, and properties of parallelograms. Visualize it using dynamic geometry software.

Using SAS and the alternate interior angle theorem, prove that a line segment joining midpoints of two sides of a triangle is parallel to and half the length of the third side. Apply this theorem to a line segment that cuts two sides of a triangle proportionally.

Generalize this theorem to prove that the figure formed by joining consecutive midpoints of sides of an arbitrary quadrilateral is a parallelogram. (This result is known as the Midpoint Quadrilateral Theorem or Varignon's Theorem.)

Use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle ($a^2 + b^2 = c^2$) and thus obtain an algebraic proof of the Pythagorean Theorem.

Prove that the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments into which its foot divides the hypotenuse.

Prove the converse of the Pythagorean Ttheorem, using the theorem itself as one step in the proof. Some students might engage in an exploration of Pythagorean Triples (e.g., 3-4-5, 5-12-13, etc.), which provides an algebraic extension and an opportunity to explore patterns.

Instructional Resources/Tools

Cardboard models of right triangles.
Dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).
Video: *The Theorem of Pythagoras* from Project MATHEMATICS!
Websites for the Pythagorean Theorem

Jim Loy's Pythagorean Theorem:
Animated proofs of the Pytharorean Theorem: Pythagorean Theorem and its Many Proofs:

Common Misconceptions

Some students may confuse the alternate interior angle theorem and its converse as well as the Pythagorean theorem and its converse.

Diverse Learners

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Specific strategies for mathematics may include:

There are approximately 400 known proofs of the Pythagorean theorem. Students can investigate some of these. One of them is attributed to James A. Garfield, an Ohio native and the 20th President of the United States.

Students can investigate the history of The Pythagorean theorem in several ancient cultures including Mesopotamia, China, and Greece.

Connections:

The Pythagorean theorem and its converse are proved and applied in Grade 8. In high school, another proof, based on similar triangles, is presented.

The alternate interior angle theorem and its converse, as well as properties of parallelograms, are established informally in Grade 8 and proved formally in high school.

Domain	**Similarity, Right Triangles, and Trigonometry**
Cluster	***Define trigonometric ratios and solve problems involving right triangles***
Standards	6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. 7. Explain and use the relationship between the sine and cosine of complementary angles. 8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Review vocabulary (opposite and adjacent sides, legs, hypotenuse and complementary angles)associated with right triangles.

Make cutouts or drawings of right triangles or manipulate them on a computer screen using dynamic geometry software and ask students to measure side lengths and compute side ratios. Observe that when triangles satisfy the AA criterion, corresponding side ratios are equal. Side ratios are given standard names, such as sine, cosine and tangent. Allow adequate time for students to discover trigonometric relationships and progress from concrete to abstract understanding of the trigonometric ratios.

Show students how to use the trigonometric function keys on a calculator. Also, show how to find the measure of an acute angle if the value of its trigonometric function is known.

Investigate sines and cosines of complementary angles, and guide students to discover that they are equal to one another. Point out to students that the “co” in cosine refers to the “sine of the complement.”

Observe that, as the size of the acute angle increases, sines and tangents increase while cosines decrease.
Stress trigonometric terminology by the history of the word “sine” and the connection between the term “tangent” in trigonometry and tangents to circles.

Have students make their own diagrams showing a right triangle with labels showing the trigonometric ratios. Although students like mnemonics such as SOHCAHTOA, these are not a substitute for conceptual understanding. Some students may investigate the reciprocals of sine, cosine, and tangent to discover the other three trigonometric functions.

Use the Pythagorean theorem to obtain exact trigonometric ratios for 30°, 45°, and 60° angles.
Use cooperative learning in small groups for discovery activities and outdoor measurement projects.

Have students work on applied problems and project, such as measuring the height of the school building or a flagpole, using clinometers and the trigonometric functions.

Instructional Resources/Tools

Cutouts of right triangles
Rulers
Protractors
Scientific calculators
Dynamic geometry software (Geometer’s Sketchpad®, Cabri®, or Geogebra®)
Trig Trainer® instructional aids
Clinometers (can be made by the students)

Websites for the history of mathematics:

Trigonometric Functions

Trigonometric Course

Video: *Sines and Cosines, Part II* from Project MATHEMATICS!

Common Misconceptions

Some students believe that right triangles must be oriented a particular way.

Some students do not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle.

Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.

Diverse Learners

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Specific strategies for mathematics may include:

Students may investigate the history of trigonometry in general and the word “sine” in particular.

Invite engineers or other professionals who use trigonometry to speak to the class.

Connections:

Trigonometry is not introduced until high school. Right triangle trigonometry (a geometry topic) has implications when studying algebra and functions. For example, trigonometric ratios are functions of the size of an angle, the trigonometric functions can be revisited after radian measure has been studied, and the Pythagorean theorem can be used to show that $(\sin A)^2 + (\cos A)^2 = 1$.

Domain	Similarity, Right Triangles, and Trigonometry
Cluster	*Apply trigonometry to general triangles*
Standards	9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. 11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, and goes beyond the mathematics that all students should study in order to be college- and career-ready:

Extend the definitions of sine and cosine to functions of obtuse angles.

The formula Area = ½ ab sin(C) can be derived from the definition of sine and the formula for the area of a triangle. The formulas Area = ½ ac sin(B) and Area = ½ bc sin(A) are also valid. The Law of Sines follows directly from equating these three formulas for the area of a given triangle.

There are several proofs of the Law of Cosines. One of the easiest to follow uses the Pythagorean theorem.

Dynamic geometry software can be used to show how the law of cosines generalizes the Pythagorean theorem by moving one vertex of a triangle to make the angle acute, right or obtuse.

Given a triangle in which the measures of three parts (at least one of which is a side) are known, the Law of Sines or the Law of Cosines can be used to find the measures of the remaining three parts. This procedure is known as “solving the triangle” or “triangulation.” Triangulation is an important tool used by surveyors for determining the location of a point by measuring angles to it from known points at either end of a fixed baseline, rather than directly measuring distances to the point.

The Laws of Sines and Cosines can be applied to other problems in geometry, such as finding the perimeter of a parallelogram if the lengths of the diagonals and the angle at which the diagonals intersect are known.

The Law of Cosines can be used in finding the vector sum or difference of two given vectors. A physical application is finding the resultant of two forces.

Instructional Resources/Tools

Scientific calculator

Dynamic geometry software (Geometer’s Sketchpad®, Cabri®, or Geogebra®)

Real-world problems that involve solving a triangle by use of Pythagorean Theorem, right triangle trigonometry and/or the Law of Sines and the Law of Cosines

Common Misconceptions

Some students may think that definitions of sine, cosine and tangent using right triangles ratios are valid for solving any triangles. In reality, these ratios only applicable to right triangles, necessitating the use of the Laws of Sines and Cosines.

When applying the Law of Sines, there is an ambiguous case (SSA) in which there are two different possible solutions for the third side of the triangle.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

The Law of Cosines can be applied in the study of vectors. The Law of Cosines is a generalization of the Pythagorean Theorem. The Pythagorean Theorem was first studied in Grade 8.

Domain	Circles
Cluster	***Understand and apply theorems about circles***
Standards	1. Prove that all circles are similar. 2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* 3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. 4. (+) Construct a tangent line from a point outside a given circle to the circle.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Given any two circles in a plane, show that they are related by dilation. Guide students to discover the center and scale factor of this dilation and make a conjecture about all dilations of circles.

Starting with the special case of an angle inscribed in a semicircle, use the fact that the angle sum of a triangle is 180° to show that this angle is a right angle. Using dynamic geometry, students can grab a point on a circle and move it to see that the measure of the inscribed angle passing through the endpoints of a diameter is always 90°. Then extend the result to any inscribed angles. For inscribed angles, proofs can be based on the fact that the measure of an exterior angle of a triangle equals the sum of the measures of the nonadjacent angles. Consider cases of acute or abtuse inscribed angles.

Use properties of congruent triangles and perpendicular lines to prove theorems about diameters, radii, chords, and tangent lines.

Use formal geometric constructions to construct perpendicular bisectors of the sides and angle bisectors of a given triangle. Their intersections are the centers of the circumscribed and inscribed circles, respectively.

Dissect an inscribed quadrilateral into triangles, and use theorems about triangles to prove properties of these quadrilaterals and their angles.

Constructing tangents to a circle from a point outside the circle is a useful application of the result that an angle inscribed in a semicircle is a right angle.

Instructional Resources/Tools

Ruler
Compass
Protractor
Computer dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®).

Common Misconceptions

Students sometimes confuse inscribed angles and central angles.

Students may think they can tell by inspection whether a line intersects a circle in exactly one point. It may be beneficial to formally define a tangent line as the line perpendicular to a radius at the point where the radius intersects the circle.

Diverse Learners
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Specific strategies for mathematics may include:

Challenge students to generalize the results about angle sums of triangles and quadrilaterals to a corresponding result for *n*-gons.

Connections: Constructing inscribed and circumscribed circles of a triangle is an application of the formal constructions studied in G – CO.12.

Domain	Circles
Cluster	***Find arc lengths and areas of sectors of circles***
Standards	5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Begin by calculating lengths of arcs that are simple fractional parts of a circle (e.g. 1/6), and do this for circles of various radii so that students discover a proportionality relationship.

Provide plenty of practice in assigning radian measure to angles that are simple fractional parts of a straight angle.

Stress the definition of radian by considering a central angle whose intercepted arc has its length equal to the radius, making the constant of proportionality 1. Students who are having difficulty understanding radians may benefit from constructing cardboard sectors whose angles are one radian. Use a ruler and string to approximate such an angle.

Compute areas of sectors by first considering them as fractional parts of a circle. Then, using proportionality, derive a formula for their area in terms of radius and central angle. Do this for angles that are measured both in degrees and radians and note that the formula is much simpler when the angels are measured in radians.

Derive formulas that relate degrees and radians.

Introduce arc measures that are equal to the) measures of the intercepted central angles in degrees or radians..

Emphasize appropriate use of terms, such as, angle, arc, radian, degree, and sector.

Instructional Resources/Tools

Ruler
Compass
Protractor
String
Computer dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®)
Video: *The Story of Pi* from Project MATHEMATICS!

Common Misconceptions

Sectors and segments are often used interchangeably in everyday conversation. Care should be taken to distinguish these two geometric concepts.

The formulas for converting radians to degrees and vice versa are easily confused. Knowing that the degree measure of a given angle is always a number larger than the radian measure can help students use the correct unit.

Diverse Learners

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<u>Specific strategies for mathematics may include:</u>
Students may create intricate designs based on sectors of circles as an art project.

<u>Connections:</u> Formulas for area and circumference of a circle were developed in grade 7. In this cluster the formulas are generalized to fractional parts of a circle and will prepare students for the study of trigonometry

Domain	Expressing Geometric Properties with Equations
Cluster	***Translate between the geometric description and the equation for a conic section***
Standards	1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 2. Derive the equation of a parabola given a focus and directrix. 3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Review the definition of a circle as a set of points whose distance from a fixed point is constant.

Review the algebraic method of completing the square and demonstrate it geometrically.

Illustrate conic sections geometrically as cross sections of a cone.

Use the Pythagorean theorem to derive the distance formula. Then, use the distance formula to derive the equation of a circle with a given center and radius, beginning with the case where the center is the origin.
Starting with any quadratic equation in two variables (x and y) in which the coefficients of the quadratic terms are equal, complete the squares in both x and y and obtain the equation of a circle in standard form.

Given two points, find the equation of the circle passing through one of the points and having the other as its center.

Define a parabola as a set of points satisfying the condition that their distance from a fixed point (focus) equals their distance from a fixed line (directrix). Start with a horizontal directrix and a focus on the y-axis, and use the distance formula to obtain an equation of the resulting parabola in terms of y and x^2. Next use a vertical directrix and a focus on the x-axis to obtain an equation of a parabola in terms of x and y^2. Make generalizations in which the focus may be any point, but the directrix is still either horizontal or vertical. Allow sufficient time for students to become familiar with new vocabulary and notation.

Given y as a quadratic equation of x (or x as a quadratic function of y), complete the square to obtain an equation of a parabola in standard form.

Identify the vertex of a parabola when its equation is in standard form and show that the vertex is halfway between the focus and directrix.

Investigate practical applications of parabolas and paraboloids.

Information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, and goes beyond the mathematics that all students should study in order to be college- and career-ready:

Students preparing for advanced courses may use the distance formula and relevant focus-directrix definitions to derive equations of ellipses and hyperbolas whose major axes are either horizontal or vertical. Encourage students to explore conic sections using dynamic geometry software.

Instructional Resources/Tools

Physical models of cones sliced to show cross sections that are circles, ellipses parabolas and hyperbolas
Dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®)
Parabolic reflectors to illustrate practical applications of parabolas

Common Misconceptions

Because new vocabulary is being introduced in this cluster, remembering the names of the conic sections can be problematic for some students.

The Euclidean distance formula involves squared, subscripted variables whose differences are added. The notation and multiplicity of steps can be a serious stumbling block for some students.

The method of completing the square is a multi-step process that takes time to assimilate. A geometric demonstration of completing the square can be helpful in promoting conceptual understanding.

Diverse Learners

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Specific strategies for mathematics may include:

Students may construct envelopes of tangents to conic sections by paper folding.

Students may investigate the reflection property of a parabola (rays parallel to the axis are reflected to pass through the focus and vice versa) and study applications such as headlight reflectors or "dish" antennas.

Students may investigate the history of conic sections, dating back to the ancient Greeks, and learn the origin of their names.

Connections:

In Grade 8 the Pythagorean theorem was applied to find the distance between two particular points. In high school, the application is generalized to obtain formulas related to conic sections.

Quadratic functions and the method of completing the square are studied in the domain of interpreting functions. The methods are applied here to transform a quadratic equation representing a conic section into standard form.

High School Conceptual Category: Geometry

Domain	**Expressing Geometric Properties with Equations**
Cluster	***Use coordinates to prove simple geometric theorems algebraically***
Standards	4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* 5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). 6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. 7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Review the concept of slope as the rate of change of the *y*-coordinate with respect to the *x*-coordinate for a point moving along a line, and derive the slope formula.

Use similar triangles to show that every nonvertical line has a constant slope.

Review the point-slope, slope-intercept and standard forms for equations of lines.

Investigate pairs of lines that are known to be parallel or perpendicular to each other and discover that their slopes are either equal or have a product of –1, respectively.

Pay special attention to the slope of a line and its applications in analyzing properties of lines.

Allow adequate time for students to become familiar with slopes and equations of lines and methods of computing them.

Use slopes and the Euclidean distance formula to solve problems about figures in the coordinate plane such as:

- Given three points, are they vertices of an isosceles, equilateral, or right triangle?
- Given four points, are they vertices of a parallelogram, a rectangle, a rhombus, or a square?
- Given the equation of a circle and a point, does the point lie outside, inside, or on the circle?
- Given the equation of a circle and a point on it, find an equation of the line tangent to the circle at that point.
- Given a line and a point not on it, find an equation of the line through the point that is parallel to the given line.
- Given a line and a point not on it, find an equation of the line through the point that is perpendicular to the given line.
- Given the equations of two non-parallel lines, find their point of intersection.

Given two points, use the distance formula to find the coordinates of the point halfway between them. Generalize this for two arbitrary points to derive the midpoint formula.

Use linear interpolation to generalize the midpoint formula and find the point that partitions a line segment in any specified ratio.

Use the distance formula to find the length of each side of a polygon whose vertices are known, and compute the perimeter of that figure.

Given the vertices of a triangle or a parallelogram, find the equation of a line containing the altitude to a specified base and the point of intersection of the altitude and the base. Use the distance formula to find the length of that altitude and base, and then compute the area of the figure.

Instructional Resources/Tools

Graph paper
Scientific or graphing calculators
Dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®)

Common Misconceptions

Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is "no slope," which is incorrect.

Diverse Learners

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Specific strategies for mathematics may include:

As students become proficient in using slopes and the distance formula to solve the kinds of problems suggested in this cluster, allow them to solve more complex problems with the aid of dynamic geometric software.

Connections:

Rates of change and graphs of linear equations were studied in Grade 8 and generalized in the Functions and Geometry Conceptual Categories in high schol. Therefore, an alternative way to define the slope of a line is to call it the tangent of an angle of inclination of the line.

In calculus, the concept of slope will be extended again to the slope of a curve at a particular point.

Domain	Geometric Measurement and Dimension
Cluster	*Explain volume formulas and use them to solve problems*
Standards	1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* 2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Revisit formulas $C = \pi d$ and $C = 2\pi r$. . Observe that the circumference is a little more than three times the diameter of the circle. Briefly discuss the history of this number and attempts to compute its value.

Use alternative ways to derive the formula for the area of the circle $A = \pi r^2$. For example,
Cut a cardboard circular disk into 6 congruent sectors and rearrange the pieces to form a shape that looks like a parallelogram with two scalloped edges. Repeat the process with 12 sectors and note how the edges of the parallelogram look "straighter." Discuss what would happen in the case as the number of sectors becomes infinitely large. Then calculate the area of a parallelogram with base ½ *C* and altitude *r* to derive the formula $A = \pi r^2$.

Wind a piece of string or rope to form a circular disk and cut it along a radial line. Stack the pieces to form a triangular shape with base *C* and altitude *r*. Again discuss what would happen if the string became thinner and thinner so that the number of pieces in the stack became infinitely large. Then calculate the area of the triangle to derive the formula $A = \pi r^2$.

Introduce Cavalieri's principle using a concrete model, such as a deck of cards. Use Cavalieri's principle with cross sections of cylinders, pyramids, and cones to justify their volume formulas.

For pyramids and cones, the factor $\frac{1}{3}$ will need some explanation. An informal demonstration can be done using a volume relationship set of plastic shapes that permit one to pour liquid or sand from one shape into another. Another way to do this for pyramids is with Geoblocks®. The set includes three pyramids with equal bases and altitudes that will stack to form a cube. An algebraic approach involves the formula for the sum of squares ($1^2 + 2^2 + \ldots + n^2$).

After the coefficient 1/3 has been justified for the formula of the volume of the pyramid ($A = \frac{1}{3}Bh$), one can argue that it must also apply to the formula of the volume of the cone by considering a cone to be a pyra*Type equation here.*mid that has a base with infinitely many sides.

The formulas for volumes of cylinders, pyramids, cones and spheres can be applied to a wide variety of problems such as finding the capacity of a pipeline; comparing the amount of food in cans of various shapes; comparing capacities of cylindrical, conical and spherical storage tanks; using pyramids and cones in architecture; etc. Use a combination of concrete models and formal reasoning to develop conceptual understanding of the volume formulas.

Some information below includes additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics, and goes beyond the mathematics that all students should study in order to be college- and career-ready:

Cavalieri's principle can also be applied to obtain the volume of a sphere, using an argument similar to that employed by Archimedes more than 2000 years ago. In this demonstration, cross sections of a sphere of radius *R* and a cone having radius 2*R* and altitude 2*R* are balanced against cross sections of a cylinder having radius 2*R* and altitude 2*R*. (Details are shown on the Archimedes website listed below.)

Instructional Resources/Tools

Rope or string
Concrete models of circles cut into sectors and cylinders, pyramids, cones and spheres cut into slices.
Rope or string
Geoblocks® or comparable models of solid shapes
Volume relationship set of plastic shapes
Web sites that explore volumes of solids Mathman,
Web site on Archimedes and the volume of a sphere: **http://physics.weber.edu/carroll/archimedes/method1.htm**
Dynamic geometry software (Geometer's Sketchpad®, Cabri®, or Geogebra®)
Video: *The Story of Pi* from Project MATHEMATICS!

Common Misconceptions

An informal survey of students from elementary school through college showed the number pi to be the mathematical idea about which more students were curious than any other. There are at least three facets to this curiosity: the symbol π itself, the number 3.14159…, and the formula for the area of a circle. All of these facets can be addressed here, at least briefly.

Many students want to think of infinity as a number. Avoid this by talking about a quantity that becomes larger and larger with no upper bound.

The inclusion of the coefficient $\frac{1}{3}$ in the formulas for the volume of a pyramid or cone and $\frac{4}{3}$ in the formula for the volume of a sphere remains a mystery for many students. In high school, students should attain a conceptual understanding of where these coefficient come from. Concrete demonstrations, such as pouring water from one shape into another should be followed by more formal reasoning.

Diverse Learners

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Specific strategies for mathematics may include:
Invite an architect or an engineer to visit the class to demonstrate some uses of cylinders, pyramids, cones or spheres in their work.

Students may investigate the history of pi. There are several major questions to be answered.

(1) When and why was the symbol π chosen to represent this number?
(2) How did the "formula" for the area of a circle evolve?
(3) How is it possible to compute more than a billion digits of the number pi?
(4) What is meant by saying that pi is a transcendental number?

Connections:

In Grade 8, students were required to know and use the formulas for volumes of cylinders, cones, and spheres. In this cluster those formulas are derived by a combination of concrete demonstrations and formal reasoning.

Students who eventually study calculus will formally derive formulas for the volume of a pyramid, cone and sphere using definite integrals.

Domain	**Geometric Measurement and Dimension**
Cluster	***Visualize relationships between two-dimensional and three-dimensional objects***
Standards	4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Review vocabulary for names of solids (e.g., right prism, cylinder, cone, sphere, etc.).

Slice various solids to illustrate their cross sections. For example, cross sections of a cube can be triangles, quadrilaterals or hexagons. Rubber bands may also be stretched around a solid to show a cross section.

Cut a half-inch slit in the end of a drinking straw, and insert a cardboard cutout shape. Rotate the straw and observe the three-dimensional solid of revolution generated by the two-dimensional cutout.

Java applets on some web sites can also be used to illustrate cross sections or solids of revolution.

Encourage students to create three-dimensional models to be sliced and cardboard cutouts to be rotated. Students can also make three-dimensional models out of modeling clay and slice through them with a plastic knife.

Instructional Resources/Tools

Concrete models of solids such as cubes, pyramids, cylinders, and spheres. Include some models that can be sliced, such as those made from Styrofoam

Rubber bands.

Cardboard cutouts of 2-D figures (e.g. rectangles, triangles, circles)

Drinking straws

Web sites, that illustrate geometric models. Some examples are:

- The Geometry Junkyard
- Wolfram Mathworld

Web sites that can be used to create solids of revolution. An example is:

- The Lathe

Common Misconceptions

Some cross sections are more difficult to visualize than others. For example, it is often easier to visualize a rectangular cross section of a cube than a hexagonal cross section.

Generating solids of revolution involves motion and is difficult to visualize by merely looking at drawings.

Diverse Learners

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Specific strategies for mathematics may include:

Students who have artistic skills may want to make drawings of solids with highlighted cross sections or drawings of solids generated by rotation two-dimensional shapes.

Connections:
Slices of rectangular prisms and pyramids were explored in Grade 7. In high school, the concept is extended to a wider class of solids.

Students who eventually take calculus will learn how to compute volumes of solids of revolution by a method involving cross-sectional disks.

High School Conceptual Category: Geometry

Domain	Modeling with Geometry
Cluster	*Apply geometric concepts in modeling situations*
Standards	1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Genuine mathematical modeling typically involves more than one conceptual category. For example, modeling a herd of wild animals may involve geometry, measurement, proportional reasoning, estimation, probability and statistics, functions, and algebra. It would be somewhat misleading to try to teach a unit with the title of "modeling with geometry." Instead, these standards can be woven into other content clusters.

A challenge for teaching modeling is finding problems that are interesting and relevant to high school students and, at the same time, solvable with the mathematical tools at the students' disposal. The resources listed below are a beginning for addressing this difficulty.

Instructional Resources/Tools

A Sourcebook of Applications of School Mathematics, compiled by a Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics (1980).
Mathematics: Modeling our World, Course 1 and Course 2, by the Consortium for Mathematics and its Applications (COMAP), http://www.comap.com/.
Geometry & its Applications (GeoMAP). An exciting National Science Foundation project to introduce new discoveries and real-world applications of geometry to high school students. Produced by COMAP.
Measurement in School Mathematics, NCTM 1976 Yearbook.

Common Misconceptions

When students ask to see "useful" mathematics, what they often mean is, "Show me how to use this mathematical concept or skill to solve the homework problems." Mathematical modeling, on the other hand, involves solving problems in which the path to the solution is not obvious. Geometry may be one of several tools that can be used.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

Modeling activities are a good way to show connections among various branches of mathematics.